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Topological Symmetry of the Bosonic String

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We exhibit the topological symmetry of the bosonic string in the framework of the BRST formalism. To get the Slavnov–Taylor symmetry independent of the diffeomorphism one, we extend the latter by introducing an antiderivation. Then on the functional space, we establish that the antiderivation, the Slavnov–Taylor, and the extended Ward operators generate a supersymmetric invariance of the bosonic string.

1. INTRODUCTION

Topological field theories form a class of gauge models with the peculiarity that their observables are of topological nature, for instance, knot and link invariants in the case of three-dimensional Chern–Simons theory [1], Donaldson invariants for the four-dimensional topological Yang–Mills models [2], and many other examples [3]. Indeed, expectation values of physical observables do not vary under smooth deformations of the metric that one puts on the base manifold, and under deformations of the coupling constants that appear in the theory [3, 4]. In fact, in their original formulation, topological field theories were constructed to have the global symmetry that arises as the BRST symmetry of an appropriate quantum field theory with different gauge choices, and then to be specific gauge fixings for a higher theory.

Topological field models are classified into two types:

1. Witten-type theories, for which the complete quantum action S_q, the classical action plus all necessary gauge-fixing and ghost terms, is BRST-exact, i.e.,

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$$S_q = \{Q, V\} \tag{1.1}$$

for some functional $V(\Phi, g)$ of the fields, and Q is the nilpotent (and, in general, metric independent) BRST charge [2]. For every field there is a superpartner with the same spin, but with opposite Grassmann charge.

2. Schwarz-type topological models, which are characterized by a metric-independent, nontrivial, classical action $S_C(\Phi)$. Upon gauge fixing, the total action (in certain cases) takes the form

$$S_q(\Phi, g) = S_C(\Phi) + \{Q, V(\Phi, g)\} \tag{1.2}$$

As the energy-momentum tensor $T_{\alpha\beta}$ is defined by the change in the action under an infinitesimal deformation of the metric

$$\delta_g S_q = 1/2 \int_\Sigma d^m x \sqrt{g}\delta g^{\alpha\beta} T_{\alpha\beta} \tag{1.3}$$

from equations (1.1) and (1.2) we get for the energy-momentum tensor the following expression for the two types of topological models:

$$T_{\alpha\beta} = \left\{Q, 2/\sqrt{g}\,\frac{\delta V}{\delta g^{\alpha\beta}}\right\} \tag{1.4}$$

In particular, a quantum field theory constructed from a Riemann surface Σ alone, using neither complex structure nor metric, is a topological quantum field theory. Without a metric there are no distance measurements or forces and so no conventional dynamics. The Hamiltonian of the theory has only zero eigenstates. However, the nontriviality of the model is reflected in the existence of tunneling between vacua.

String theory (and its supersymmetric version) has led to a large number of fruitful applications such as developments in conformal field theories and in lower dimensional gravity [5].

In analogy with topological field theories, the bosonic string is considered as a gauge theory for the group (*Diff* $\times$ *Weyl*), *Diff* and *Weyl* denoting, respectively, the diffeomorphisms and the Weyl-scale transformations.

In this paper, we exhibit the topological nature of the bosonic string first-quantized, as a gauge theory, in the framework of the BRST formalism. Indeed, we show that this symmetry is apparent, in the conformal gauge, at the level of separation of the Slavnov–Taylor symmetry from the diffeomorphism one. We establish that the generators of this topological symmetry are identified with the linearized Slavnov–Taylor operator, the representative, on the functional space, of an antiderivation, and with the Ward operator of extended diffeomorphisims. Then we recover the supersymmetric algebra introduced in ref. 6 by considering the z and $\bar{z}$ components of the above operators.

The so-called Beltrami parametrization of the two-dimensional world sheet metric of the bosonic string introduced in refs. 7 and 8 turns out to be quite useful for describing the string properties within the perturbative field-theoretical framework. It allows the use of a quantization procedure completely analogous to the Yang–Mills theories. Moreover, the Beltrami parametrization is the most natural parametrization which exhibits the holomorphic factorization of the Green functions with insertion of the energy-momentum tensor, according to the Belavin–Polyakov–Zamolodchikov scheme [9]. Indeed, the Beltrami parameter can be seen as the classical source for the $(T_{zz}, T_{\bar{z}\bar{z}})$ components of the energy-momentum tensor. It turns out that the Slavnov–Taylor identity corresponding to the BRST invariance of the theory can be taken as the starting point for algebraic characterization of the energy-momentum current algebra.

Also, the use of the Beltrami parameter allows one to eliminate from the very beginning the Weyl symmetry degree of feedom, the remaining diffeomorphism transformations being kept as the basic local invariance of the string action [7]. However, it is well known that diffeomorphism invariance cannot be preserved at the quantum level. There is indeed an anomaly whose numerical coefficient is nonzero, and its presence implies the existence of unphysical negative norm states in the Fock space of the string excitations [5]. A consistent theory requires the vanishing of this coefficient, which implies that the target space of the bosonic string is a 26-dimensional space-time [10].

2. THE BELTRAMI PARAMETRIZATION OF THE BOSONIC STRING

In its Euclidean version the bosonic string is described by the classical action

$$S_{inv}(X, g) = 1/2 \int_{\Sigma} d^2x \, \sqrt{g} g^{\alpha\beta} \, \partial_\alpha X \partial_\beta X \tag{2.1}$$

where $g_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$ is a metric on the two-dimensional Riemannian manifold Σ that is the string world sheet. $X = (X^i, i = 1, \ldots, D)$ are the string coordinates which map Σ into the D-dimensional flat space R^D; X^i: $\Sigma \to R^D$ and (x^1, x^2) are coordinates of the surface Σ.

Conformal classes of the metric on the Riemann surface Σ are parametrized by Beltrami differentials μ satisfying $\mathrm{Sup}_\Sigma|\mu| < 1$ [11–13]. If we consider a reference complex structure on Σ parametrized by a complex coordinate system $(z, \bar{z})$, the conformal class of the metric g is characterized by

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta = \rho^2 |dz + \mu d\bar{z}|^2 \tag{2.2}$$

where $\rho(z, \bar{z})$ is the conformal factor.

In this framework the action (2.1) becomes

$$S_{inv}(X, \mu, \bar{\mu}) = 1/2 \int_\Sigma dm \left(\frac{1}{1 - \mu\bar{\mu}}\right)(\partial - \bar{\mu}\bar{\partial})X(\bar{\partial} - \mu\partial)X \tag{2.3}$$

with

$$dm \equiv d\bar{z} \wedge dz/2i, \qquad \partial \equiv \partial/\partial z, \quad \bar{\partial} \equiv \partial/\partial\bar{z}$$

As mentioned in the Introduction and as one can observe, the scaling factor of (2.2) has been eliminated from the new form (2.3) of the action. Indeed, due to the Weyl invariance $[g(x) \to e^{\varphi(x)} g(x)]$, S_{inv} depends only on a conformal class of the metric that is parametrized by the Beltrami differential μ. Also, the action (2.3) describes a conformal theory of a scalar field X that is R^D-valued and is invariant under the group $Diff_0(\Sigma)$, the group of the diffeomorphisms on the Riemann surface Σ, which are related to the identity [11]. This invariance is expressed in the BRST formulation by the structure equations

$$\begin{aligned} sX &= (c \cdot \partial)X \\ s\mu &= (\bar{\partial} - \mu\partial + \partial\mu)C \\ sC &= C\partial C \end{aligned} \tag{2.4}$$

and

$$\text{c.c.}$$

with

$$\begin{aligned} s^2 &= 0 \\ C &= c + \mu\bar{c} \end{aligned} \tag{2.5}$$

$(c, \bar{c})$ are the anticommuting ghost variables corresponding to two diffeomorphism variables. At this level, the exterior fields μ and $\bar{\mu}$ are considered as quantum fields coupled to the string quantum field X and describe all possible analytic structures defined on Σ on which we should integrate to get the Polyakov formulation of string theories [10].

In order to fix the gauge, we introduce a pair of antighosts $(b, \bar{b})$ and of Lagrange multipliers $(\beta, \bar{\beta})$ transforming under BRST as

$$sb = \beta \tag{2.6}$$
$$s\beta = 0$$

and

c.c.

In the Landau conformal gauge, i.e.,

$$\mu = \mu_0 \tag{2.7}$$

where μ_0 is a classical prescribed Beltrami differential left invariant by the BRST operator, that is,

$$s\mu_0 = 0 \tag{2.8}$$

the gauge-fixing action reads

$$S_{gf}(\mu, b, C, \mu_0, \beta) = \int_\Sigma dm\, [-\beta(\mu - \mu_0) + bs\mu + \text{c.c.}] \tag{2.9}$$

Then, one can verify that the effective action defined by

$$S_{eff} = S_{inv} + S_{gf} \tag{2.10}$$

is BRST-invariant:

$$sS_{eff} = 0 \tag{2.11}$$

Moreover, if the auxiliary field β is eliminated by its equation of motion (2.7), which amounts to replacing everywhere the original Beltrami parameter with the classical one μ_0 (which will be considered next), Eq. (2.11) holds such that

$$sb = 0 \tag{2.12}$$

As usual, coupling the nonlinear BRST variations of $(X, C, \overline{C})$ to invariant external sources $(X_s, C_s, \overline{C}_s)$

$$S_{ext} = \int_\Sigma dm\, (X_s sX + C_s sC + \overline{C}_s s\overline{C}) \tag{2.13}$$

and using the algebraic property

$$s\mu = \delta S_{eff}/\delta b \tag{2.14}$$

gives the Slavnov–Taylor identity

$$S_1 S_{tot} = 0 \tag{2.15}$$

where $S_{tot} = S_{eff} + S_{ext}$ is the total classical action and S_1 is the extended BRST operator to the sources and represents the Slavnov–Taylor operator:

$$S_1 = \int_\Sigma dm \left(\frac{\delta S_{tot}}{\delta X_s} \frac{\delta}{\delta X} + \frac{\delta S_{tot}}{\delta X} \frac{\delta}{\delta X_s} + \frac{\delta S_{tot}}{\delta \mu} \frac{\delta}{\delta b} + \frac{\delta S_{tot}}{\delta b} \frac{\delta}{\delta \mu} \right.$$
$$\left. + \frac{\delta S_{tot}}{\delta C_s} \frac{\delta}{\delta C} + \frac{\delta S_{tot}}{\delta C} \frac{\delta}{\delta C_s} + \text{c.c.} \right) \tag{2.16}$$

One can verify that S_1 is nilpotent

$$S_1^2 = 0 \tag{2.17}$$

and its action on the fields and on the sources is given by

$$S_1 X = sX = \frac{\delta S_{ext}}{\delta X_s}$$
$$S_1 X_s = \partial_\alpha (c^\alpha X_s) = \frac{\delta S_{ext}}{\delta X} \tag{2.18}$$
$$S_1 C = \frac{\delta S_{tot}}{\delta C_s} = C \partial C$$
$$S_1 C_s = \frac{\delta S_{tot}}{\delta C} = (C \partial C + 2 \partial C) C_s + (\bar{\partial} - \mu \partial - 2 \partial \mu) b - X_s \frac{\partial - \bar{\mu} \bar{\partial}}{1 - \mu \bar{\mu}} X$$
$$S_1 b = \frac{\delta(S_{gf} + S_{ext})}{\delta \mu} = (C \partial C + 2 \partial C) b + \frac{\bar{C} - \bar{\mu} C}{1 - \mu \bar{\mu}} X$$
$$S_1 \mu = \frac{\delta S_{tot}}{\delta b} = s\mu$$

and

$$\text{c.c.}$$

The classical total action S_{tot} can be seen as describing the propagation of the quantized fields (X, C, $\bar{C}$, b, $\bar{b}$) in a nontrivial classical background metric whose components are parametrized by μ. Moreover, the classical Slavnov–Taylor identity is taken to be the starting point of the analysis of the quantum aspects of the model.

Before going any further, let us stress that the effective action is invariant under infinitesimal diffeomorphisms whose action on the fields is given by [11]

$$\delta_\xi X = (\xi^z + \xi^{\bar{z}} \bar{\partial}) X \tag{2.19}$$
$$\delta_\xi \mu = (\bar{\partial} - \mu \partial + \partial \mu)(\xi^z + \mu \xi^{\bar{z}})$$
$$\delta_\xi C = (\xi^z \partial + \xi^{\bar{z}} \bar{\partial} - \partial \xi^z - \mu \partial \xi^{\bar{z}}) C$$

$$\delta_\xi b = (\xi^z \partial + \xi^{\bar{z}} \bar{\partial} + 2\partial\xi^z + 2\mu\partial\xi^{\bar{z}})b$$

and

$$\text{c.c.}$$

where $(\xi^z, \xi^{\bar{z}})$ are the diffeomorphism parameters and satisfy the following Lie algebra:

$$[\delta_\xi, \delta_{\xi'}] = \delta_{[\xi,\xi']_{D_0}} \tag{2.20}$$

with D_0 the Lie algebra of the group $Diff_0(\Sigma)$.

Let us stress that the closure relation

$$[\delta_\xi, S_1] = 0 \tag{2.21}$$

is not satisfied on all the fields:

$$[\delta_\xi, S_1]b = 2\partial\xi^{\bar{z}} b s\mu \tag{2.22}$$

and the diffeomorphism variations of the BRST sources X_s and C_s are not defined. Then, the two symmetries characterizing the classical total action cannot be considered independent at the quantum level. To overcome this problem, we have to replace the diffeomorphism δ_ξ by another extended one (to the sources), Δ_ξ defined by

$$\begin{aligned}
\Delta_\xi X &= \delta_\xi X \\
\Delta_\xi X_s &= \partial_\alpha(\xi^\alpha X_s) \\
\Delta_\xi \mu &= \delta_\xi \mu \\
\Delta_\xi C &= \delta_\xi C \\
\Delta_\xi b &= \delta_\xi b - \partial\xi^{\bar{z}} C_s C \\
\Delta_\xi C_s &= \partial_\alpha(\xi^\alpha C_s) + C_s(\partial\xi^z + \mu\partial\xi^{\bar{z}})
\end{aligned} \tag{2.23a}$$

and

$$\text{c.c.}$$

in order to have

$$[S_1, \Delta_\xi] = 0 \tag{2.23b}$$

One can verify that the diffeomorphism Δ_ξ can be expressed as the anticommutator

$$\Delta_\xi = \{S_1, \hat{i}_\xi\}, \qquad \xi \in D_0 \tag{2.24}$$

where S_1 is the Slavnov–Taylor operator and $\hat{i}_\xi$ is the antiderivation defined by its action on the fields as follows:

$$\begin{aligned} \hat{i}_\xi c &= \xi \\ \hat{i}_\xi C &= \xi^z + \mu\xi^{\bar{z}} \\ \hat{i}_\xi b &= \xi^{\bar{z}} C_s \\ \hat{i}_\xi(\mu, X, X_s, C_s) &= 0 \end{aligned} \tag{2.25}$$

For $\xi, \xi' \in D_0$, $\hat{i}_\xi$ satisfies the anticommutation relation

$$\{\hat{i}_\xi, \hat{i}_{\xi'}\} = 0 \tag{2.26}$$

Furthermore, the antiderivation $\hat{i}_\xi$ is realized on the functional space by the functional operator

$$J_\xi = \int_\Sigma dm \left(\hat{i}_\xi C \frac{\delta}{\delta C} + \hat{i}_\xi b \frac{\delta}{\delta b} + \text{c.c.} \right) \tag{2.27}$$

For the diffeomorphism Δ_ξ, one can satisfy the following algebra:

$$[\Delta_\xi, \Delta_{\xi'}] = \Delta_{[\xi,\xi']} \tag{2.28a}$$

$$[\Delta_\xi, \hat{i}_{\xi'}] = \hat{i}_{[\xi,\xi']} \tag{2.28b}$$

with $\xi, \xi' \in D_0$.

At the functional level Δ_ξ is represented by the operator

$$W(\xi) = \int_\Sigma dm \left(\Delta_\xi \phi \frac{\delta}{\delta \phi} + \text{c.c.} \right) \tag{2.29}$$

where $\phi = (X, X_s, C, C_s, \mu, b)$, which extends the Ward operator [11] to the BRST sources, satisfies the diffeomorphism algebra

$$[W(\xi), W(\xi')] = W([\xi, \xi']_{D_0}) \tag{2.30}$$

and the extended Ward identity

$$W(\xi) S_{tot} = 0 \tag{2.31}$$

Finally, the two symmetries are independent and the total classical action satisfies the Slavnov–Taylor identity (2.15) and the extended Ward identity (2.31)).

In the next section we will establish that the operator J_ξ that was essential in extending the diffeomorphism action to the BRST sources generates a supersymmetric structure of the bosonic string, and then the topological nature of this model is apparent when the Slavnov symmetry is separated from the diffeomorphism one [when (2.23) holds].

3. TOPOLOGICAL STRUCTURE OF THE BOSONIC STRING

It is easy to show that the operator J_ξ can be rewritten as

$$J_\xi = \int_\Sigma \left\{ \xi^z \left(\overline{\mu} \frac{\delta}{\delta \overline{C}} + \frac{\delta}{\delta C} + \overline{C}_s \frac{\delta}{\delta \overline{b}} \right) + \text{c.c.} \right\} \tag{3.1}$$

Moreover, one can verify the following relation:

$$J_\xi S_{tot} = S_1 \int_\Sigma dm \; \{\xi(C_s + \overline{\mu}\overline{C}_s) + \overline{\xi}(\overline{C}_s + \mu C_s)\} \tag{3.2}$$

As the measure and the diffeomorphism parameter ξ are S_1-invariant, Eq. (3.2) can be rewritten as

$$J_\xi S_{tot} = \int_\Sigma dm \; [\xi S_1(C_s + \overline{\mu}\overline{C}_s) + \overline{\xi} S_1(\overline{C}_s + \mu C_s)] \tag{3.3}$$

Due to the holomorphic factorization apparent in Eqs. (3.1)–(3.3), which is the important property of the Beltrami parametrization, we get locally the following identities:

$$\tilde{W} S_{tot} = \tilde{\Delta} \tag{3.4}$$

and

$$\text{c.c.},$$

with

$$\tilde{W} = \overline{\mu} \frac{\delta}{\delta \overline{C}} + \frac{\delta}{\delta C} + \overline{C}_s \frac{\delta}{\delta \overline{b}} \tag{3.5}$$

and

$$\tilde{\Delta} = S_1(C_s + \overline{\mu}\overline{C}) \tag{3.6}$$

Moreover, by direct calculation one gets

$$\tilde{\Delta} = X_s \partial X + \overline{C_s} \, \partial \overline{C} + C_s \partial \overline{C} - \overline{b} \partial \overline{\mu} - b \partial \mu \tag{3.7}$$

However, the integrated expressions for $\tilde{W}$ and $\tilde{\Delta}$ are given by

$$W = \int_{\Sigma} dm \left(\overline{\mu} \frac{\delta}{\delta \overline{C}} + \frac{\delta}{\delta C} + \overline{C_s} \frac{\delta}{\delta \overline{b}} \right) \tag{3.8}$$

and

$$\Delta = S_1 \int_{\Sigma} dm \, (C_s + \overline{\mu} \overline{C}) \tag{3.9}$$

This is justified in the reference complex structure parametrized by the fixed Beltrami differential μ_0 (defining the conformal gauge), which we continue to label μ. Then, we recover the supersymmetric algebra of the bosonic string introduced in ref. 6 generated by W, $\overline{W}$, and S_1, which is expressed as

$$\{S_1, W\} = \partial \tag{3.10a}$$

$$\{S_1, \overline{W}\} = \overline{\partial} \tag{3.10b}$$

$$\{W, \overline{W}\} = \{W, W\} = \{\overline{W}, \overline{W}\} = 0 \tag{3.11}$$

$$S_1^2 = 0 \tag{3.12}$$

Furthermore, the integrated expression (3.9) is recognized as the broken topological symmetry term in the chiral sector. However, this anomaly is S_1-exact and can be reabsorbed in the total classical action.

On the other hand, one can verify that the total classical action and the energy-momentum tensor are both S_1-exact:

$$S_{tot} = S_1 \int_{\Sigma} dm \left(\frac{1}{2} X X_S - C_S C - \overline{C}_s \overline{C} \right) \tag{3.13}$$

$$T \equiv T_{ZZ} = \frac{\delta S_{tot}}{\delta \mu} = S_1 b \tag{3.14}$$

and

c.c.

Indeed, these two relations reflect the same characteristic properties of the Witten-type topological model [3].

Now, let us return to the functional operator J_ξ in order to establish the supersymmetric structure on the functional space, i.e., to express the supersymmetric algebra [Eqs. (3.10)–(3.12)] in terms of the operators J_ξ, S_1, and $W(\xi)$.

Indeed, it is easy to verify, for all fields of the model, the following relation:

$$\Delta_\xi = \{S_1, \hat{i}_\xi\} \tag{3.15}$$

Then, one can show that this equation is represented, on the functional space, by

$$W(\xi) = \{S_1, J_\xi\} \tag{3.16}$$

by using the extended Ward and Slavnov–Taylor identities.

Conversely, one can verify that the ξ and the $\bar{\xi}$ components of Eq. (3.16) imply the anticommutation relations (3.10). Moreover, the independence of the Slavnov–Taylor symmetry from the diffeomorphism one was based on the commutation relation $[\Delta_\xi, S_1] = 0$, which can be seen to be realized, on the functional space, by the equation

$$[W(\xi), S_1] = 0 \tag{3.17}$$

from which we deduce

$$[\partial, S_1] = 0 = [\bar{\partial}, S_1] \tag{3.18}$$

Then, Eq. (2.26) can be represented by the anticommutation relation

$$[J_\xi, J_{\xi'}] = 0 \tag{3.19}$$

This can be verified from the action of the operators J_ξ and $J_{\xi'}$ on the total classical action, which gives Eq. (3.11).

Moreover, the commutation relation (2.28b) is realized by

$$[W(\xi), J_{\xi'}] = J_{[\xi,\xi']_{D_0}} \tag{3.20}$$

Thus we are able to establish the functional analogue of the supersymmetric algebra, introduced in ref. 6, of the bosonic string. In other words, we have shown that this topological structure takes its origin at the gauge-fixed level and when the independence of the two symmetries of the total classical action is considered. Hence, this algebra mixes the representatives of the BRST extension to the sources, the antiderivation and the diffeomorphism generators.

4. CONCLUSION AND OPEN PROBLEMS

As can be seen from the construction developed here, the equation $[\Delta_\xi, S_1] = 0$ and its representative were the key objects to reflect the topological symmetry of the bosonic string first-quantized in the BRST formalism.

Then, the separation of the diffeomorphism symmetry from the Slavnov–Taylor one is possible in the framework of a large symmetry, that is, the topological symmetry. This idea deserves further investigation.

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